$f(x)$ is a polynomial function such that $f'(-2) = f'(3) = 0$ and $f''(x) = (7x+9)(x-3)^5$. **SCORE: ____ / 3 PTS**

For each critical number of f, determine what the Second Derivative Test tells you about that critical number.

Justify your answer very briefly. Do NOT use the First Derivative Test.

$f''(-2) > 0 \longrightarrow$ LOCAL MIN (1½)

$f''(3) = 0 \longrightarrow$ NO CONCLUSION (1½)

MUST HAVE BOTH THE REASON + THE CONCLUSION TOGETHER TO GET ANY POINTS

$f(x)$ is a continuous function with derivative $f'(x) = (3+2x)^2 x^{-\frac{1}{3}}$. **SCORE: ____ / 5 PTS**

↳ DOMAIN = $\mathbb{R}$

[a] Find the critical numbers of f. **Justify your answer very briefly.**

(1) $f'(x)$ DNE @ $x = 0 \in$ DOMAIN

(1) $f'(x) = 0$ @ $x = -\frac{3}{2} \in$ DOMAIN

$x = 0, -\frac{3}{2}$

NO POINTS IF YOU SAID "NO CONCLUSION"

[b] For each critical number of f, determine what the First Derivative Test tells you about that critical number.

Justify your answer very briefly. Do NOT use the Second Derivative Test.

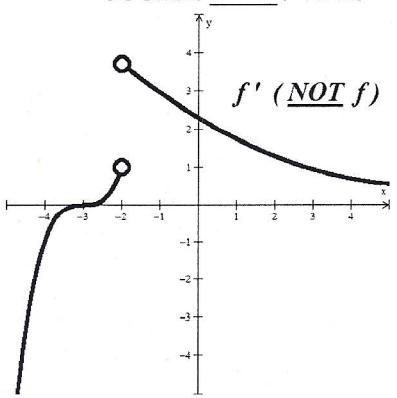
(1)

$x = 0$ LOCAL MIN (1)

$x = -\frac{3}{2}$ NOT LOCAL MIN NOR MAX (1)

$f(x)$ is a continuous function whose derivative $f'(x)$ is shown on the right. **SCORE: ____ / 4 PTS**

The following questions are about the function f, **NOT THE FUNCTION *f'***.



[a] Write "I UNDERSTAND" if you understand that the following questions are about the continuous function f, **NOT THE FUNCTION *f'***.

[b] Find the critical numbers of f.

Justify your answer very briefly.

(½) $f'(x)$ DNE @ $x = -2$

(½) $f'(x) = 0$ @ $x = -3$

[c] Find the $x-$coordinates of all inflection points of f

Justify your answer very briefly.

f' CHANGES FROM INCR TO DECR @ $x = -2$ (1½)

[d] Find all intervals over which f is increasing.

Justify your answer very briefly.

(½) $f' > 0$ ON $(-3, -2)$ (½)

AND $(-2, \infty)$ (½)

Graph $f(x) = x^4 e^{4x}$ using the process shown in lecture and in the website handout. **SCORE: _____ / 18 PTS**

Complete the table at the bottom of the page, after showing relevant work (you do NOT need to show work for entries marked ★). You will NOT receive credit for the entries in the table if the relevant work is missing.

MUST SHOW ALL USES OF L'HOSPITAL'S RULE FOR FULL (3) CREDIT

$f(0) = 0^4 e^0 = 0$

$f(x) = 0 \rightarrow x^4 = 0$ OR $e^{4x} = 0 \rightarrow x = 0$

(½) $\lim_{x\to\infty} x^4 e^{4x} = \infty$ $(\infty \cdot \infty)$

$$\lim_{x\to-\infty} x^4 e^{4x} = \lim_{x\to-\infty} \frac{x^4}{e^{-4x}} = \lim_{x\to-\infty} \frac{4x^3}{-4e^{-4x}} = \lim_{x\to-\infty} \frac{3x^2}{4e^{-4x}} = \lim_{x\to-\infty} \frac{6x}{-16e^{-4x}} = \lim_{x\to-\infty} \frac{3}{32e^{-4x}} = 0$$

$(\infty \cdot 0)$ $\left(\frac{\infty}{\infty}\right)$ $\left(\frac{-\infty}{-\infty}\right)$ $\left(\frac{\infty}{\infty}\right)$ $\left(\frac{-\infty}{-\infty}\right)$ $\left(\frac{3}{\infty}\right)$

$f'(x) = 4x^3 e^{4x} + 4x^4 e^{4x}$ EXISTS AT ALL x

$= 4x^3 e^{4x}(1+x) = 0$ @ $x = 0, -1$ (1) (½)

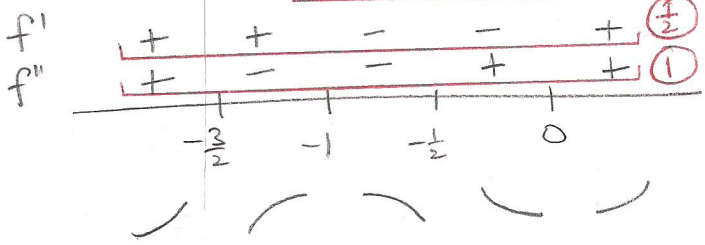
$f''(x) = 12x^2 e^{4x} + 16x^3 e^{4x} + 16x^3 e^{4x} + 16x^4 e^{4x}$ EXISTS AT ALL x

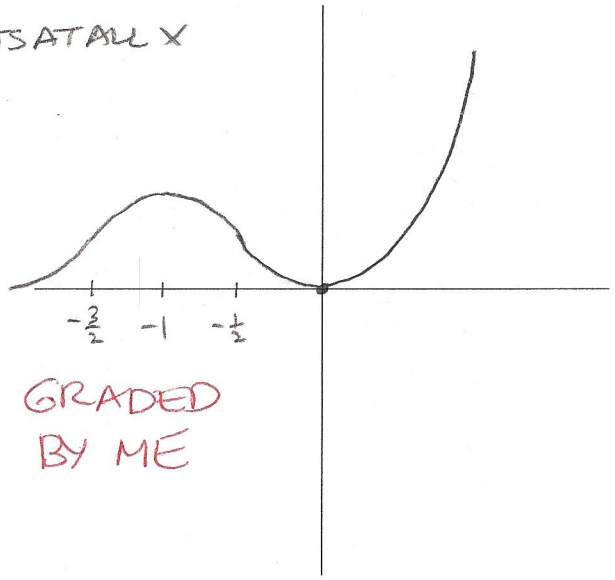
$= 12x^2 e^{4x} + 32x^3 e^{4x} + 16x^4 e^{4x}$

$= 4x^2 e^{4x}(3 + 8x + 4x^2)$

$= 4x^2 e^{4x}(3+2x)(1+2x) = 0$ (2) (½)

@ $x = 0, -\frac{3}{2}, -\frac{1}{2}$ (½)





GRADED BY ME

★ Domain	★ Discontinuities	Intercepts (specify $x-$ or $y-$)	One sided limits at each discontinuity (write using proper limit notation)	
$(-\infty, \infty)$ (½)	NONE (½)	x-INT: 0 (½) y-INT: 0 (½)	N/A	
Horizontal Asymptotes	Intervals of Increase	Intervals of Decrease	Intervals of Upward Concavity	Intervals of Downward Concavity
$y = 0$ (½)	$(-\infty, -1)$ AND $(0, \infty)$ (½)	$(-1, 0)$ (½)	$(-\infty, -\frac{3}{2})$ AND $(-\frac{1}{2}, \infty)$ (½)	$(-\frac{3}{2}, -\frac{1}{2})$ (½)
Vertical Tangent Lines	Horizontal Tangent Lines	Local Maxima	Local Minima	Inflection Points
NONE	@ $x = -1, 0$ (½)	@ $x = -1$ (½)	@ $x = 0$ (½)	@ $x = -\frac{3}{2}, -\frac{1}{2}$ (½)